Second-order topology and multidimensional topological transitions in sonic crystals

With the support by the National Natural Science Foundation of China, the research team led by Prof. Lu MingHui (卢明辉) and Prof. Chen YanFeng (陈延峰) at the National Laboratory of Solid State Microstructures, Department of Materials Science and Engineering, Nanjing University, has realized the higher-order topological insulators (HOTIs) in two-dimensional (2D) sonic crystals (SCs). This work was conducted in collaboration with the group led by Prof. Jiang JianHua (蒋建华) from Soochow University, and published in *Nature Physics* (doi: 10.1038/s41567-019-0472-1).

Topological phases of matter support gapless edge states that promise to revolutionize applications from the unique backscattering-immune electron transport to photonics and acoustics. More recently, the HOTIs, which host both gapped edge states and in-gap corner/hinge states, protected concurrently by the band topology, have been introduced, unveiling a new horizon beyond the conventional bulk-edge correspondence. However, the control and manifestation of band topology in a hierarchy of dimensions, which is at the heart of HOTIs, have not yet been witnessed and call for a decent understanding.

The quest for HOTIs is not straightforward, as the existing proposals are based on tight-binding models with complex hopping configurations, which are unlikely to be realized in conventional photonic and phononic systems. Lu, Chen and Jiang's groups teamed up and proposed a topological crystalline insulator based on sonic crystals with glide symmetries, experimentally realizing the HOTI that supports gapped edge states and in-gap corner states. Interestingly, the band topology in their system is manifested in a hierarchy of dimensions, that is, in both 2D bulk and 1D edges. Specifically, by gapping a Dirac point in 2D square-lattice SCs, a bulk topological transition occurs. Topological edge states emerge at the boundaries between the trivial and topological SCs. The underlying physics mimics the quantum spin Hall insulators, where the pseudo-Kramers degeneracy is induced by the glide symmetries and the pseudo-spins are emulated by acoustic orbital angular momenta. When the glide symmetries that induce the pseudo-Kramers degeneracy are broken at the boundaries, the edge states are gapped, giving rise to 1D massive Dirac acoustic waves. Remarkably, the Dirac masses of the 1D edge states have opposite signs for edges along the x and y directions, leading to in-gap Jackiw—Rebbi soliton states (i. e., the corner states) localized at the corners shared by those edges. Moreover, the HOTI realized in the *Nature Physics* paper allows edge topological transitions by tuning the geometry of the SCs, which, combined with the bulk topological transition, offers fascinating ways to control wave transport in multiple dimensions.

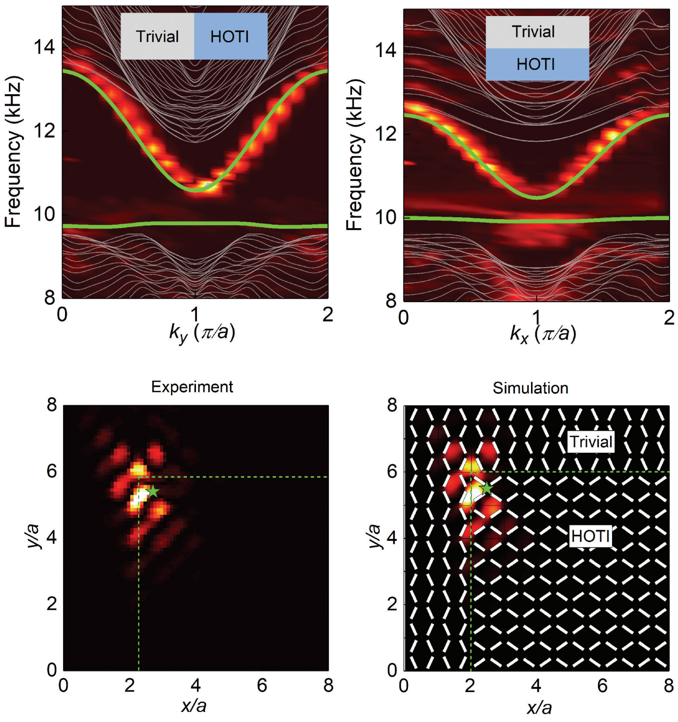

Figure Gapped edge states (upper panels) and in-gap corner states (lower panels) in the HOTI.